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# Modeling the Pulmonary Gas Exchange in Human Respiratory System

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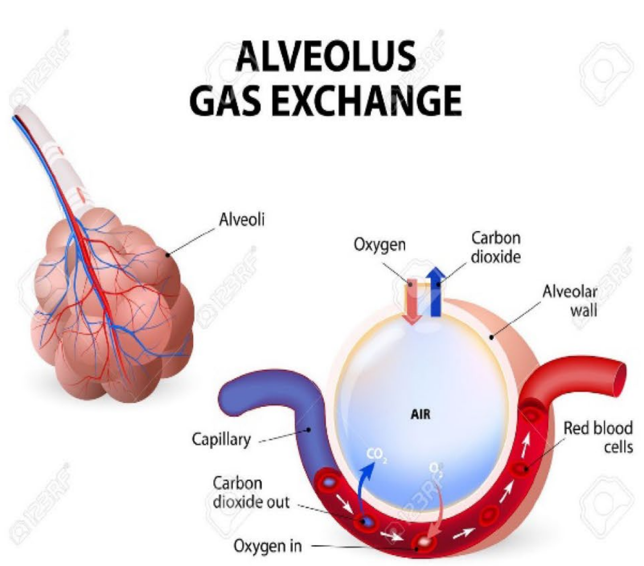
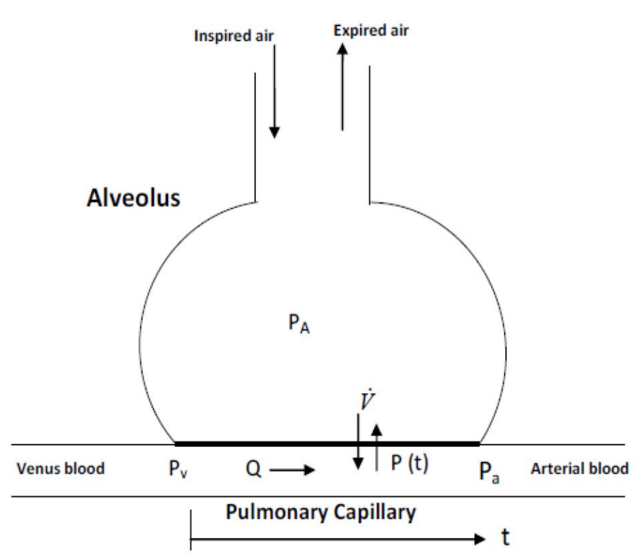
## Abstract

The human respiratory system is responsible for taking in fresh oxygen and expelling carbon dioxide. Inhaled oxygen enters alveoli which are enveloped in a network of capillaries. The diffusion of gases take place across the respiratory membrane, where the oxygen from the alveoli moves to the blood in the capillaries and carbon dioxide in the blood of capillaries moves to the alveoli. The partial pressure gradient of each gas allows them to diffuse between membranes. In this study, we explore a mathematical model for pulmonary gas exchange based on the works of Collins et al. (2015) and Tsega & Katiyas (2018)

## Background




## The Mathematical Model

Using Fick's law, the oxygen diffusion in the pulmonary membrane can be written as

$$\dot{V}(t) = D_L(P_A - P(t)) \qquad \text{(a)}$$

where $\dot{V}(t) =$ volume of oxygen transferred across the pulmonary membrane per unit time

$D_L =$ diffusion capacity of the pulmonary membrane for oxygen

$P_A =$ partial pressure of oxygen in the alveolus

$P =$ partial pressure of oxygen in the capillary

Let $V_c$ denote the total capillary blood volume and C denote the concentration of oxygen in the capillary blood. Then by applying Fick's Law for blood flow, we obtain the equation

$$\dot{V}(t) = V_c C'(t) \qquad \text{(b)}$$

## The Mathematical Model (continued)

Combining equations (a) and (b), we obtain the equation

$$C'(t) = \frac{D_L}{V_c}(P_A - P(t)) \qquad \text{(c)}$$

Applying Henry's Law to equation (c) yields the equation

$$C(t) = \alpha P(t) + \beta(Hb)(S) \qquad \text{(d)}$$

where α = the solubility of oxygen in the blood

β = the amount of oxygen per unit mass of hemoglobin when 100% saturated

Hb = the amount of hemoglobin per unit volume of blood

S = the oxygen saturation of haemoglobin in the blood.

Differentiating equations (d) with respect to t, we obtain

$$C'(t) = \alpha P'(t) + \beta(Hb)S'(t) \qquad \text{(e)}$$

The study by Collins et al (2015) relating the saturation and partial pressure from empirical data yielded the equation

$$S(P(t)) = \frac{(P(t))^3 + 150P(t)}{(P(t))^3 + 150P(t) + 23400} \qquad \text{(f)}$$

From equations (c), (e), and (f), we obtain the equation

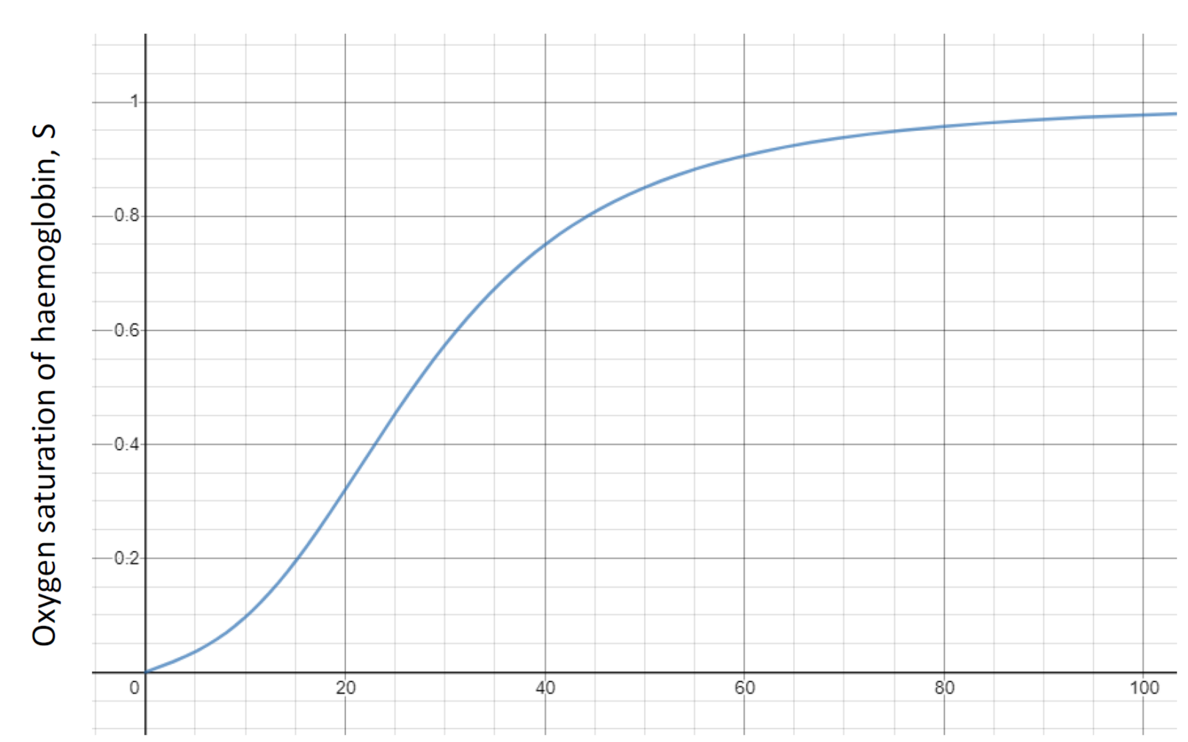
$$\left(\alpha + \frac{3580200\beta(Hb)}{((P(t))^3 + 150P(t) + 23400)^2}\right)P'(t) = \frac{D_L}{V_C}(P_A - P(t))$$

from which we get the differential equation

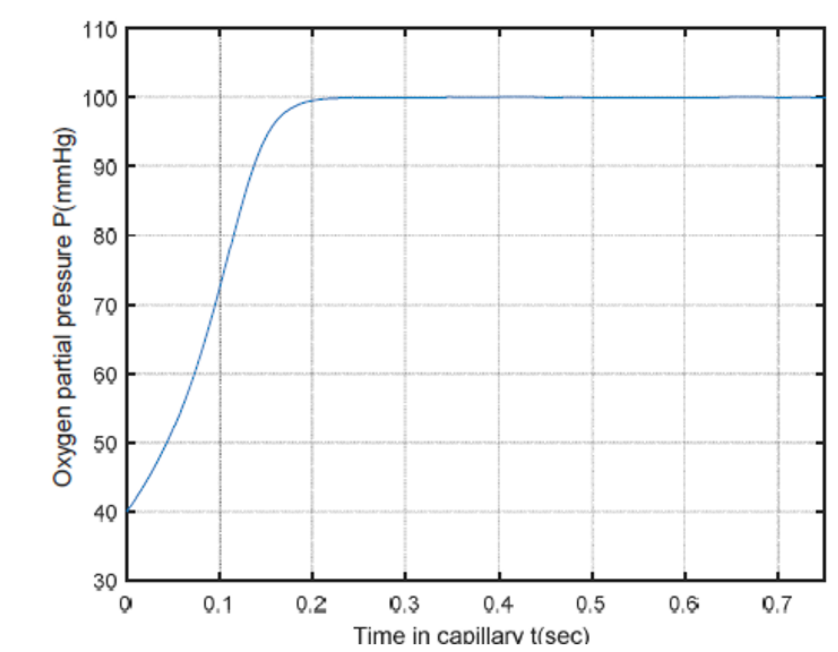
$$P'(t) = \frac{\frac{D_L}{V_C}(P_A - P(t))}{\alpha + \frac{70200\beta(Hb)(P(t)^2 + 50)}{(P(t)^3 + 150P(t) + 23400)^2}} \qquad \text{(g)}$$

| Parameter | Unit | Values |
|---|---|---|
| $P_A$ | mmHg | 100 |
| $P_V$ | mmHg | 40 |
| $D_L$ | (mL . mm) / (min . Hg) | 40 |
| $Q$ | mL /min | 6000 |
| $T$ | sec | 0.75 |
| $V_c$ | mL | 75 |
| $\alpha$ | mm/ Hg | 0.0003 |
| $Hb$ | g/mL | 0.15 |
| $\beta$ | Ml /g | 1.39 |

## Relationship between S and P from equation (f)




## Graph of Simulation Results of Solution to equation (g)




## Conclusion

- A mathematical model that relates the oxygen saturation and partial pressure of oxygen in the capillary, developed by Collins et al (2015) and Tsega & Katiya's (2018), was explored.
- The oxygen diffusion into the capillary across the pulmonary membrane depends on the diffusion capacity of the membrane for the oxygen ($D_L$) the partial pressure difference between alveolar gas and capillary blood gas ($P_A$ - P), the solubility of the oxygen in the blood (α), the capacity of haemoglobin to carry oxygen (β), and the amount of haemoglobin contained in the blood (Hb).
- The resulting differential equation was solved numerically using the Runge-Kutta Algorithm.

## References

Collins, J. A., Rudenski, A., Gibson, J., Howard, L., & O'Driscoll, R. (2015). Relating oxygen partial pressure, saturation and content: the haemoglobin–oxygen dissociation curve. *Breathe*, *11*(3), 194-201.

Martin, S., & Maury, B. (2013). Modeling of the oxygen transfer in the respiratory process∗. *ESAIM: Mathematical Modelling and Numerical Analysis*, *47*(4), 935-960.

Tsega, E. G., Katiyar, V. K., & Gupta, P. (2019). Breathing Patterns of Healthy Human Response to Different Levels of Physical Activity. *Journal of Biomedical Engineering*, *7*(1), 1-4.

Whiteley, J. P., Gavaghan, D. J., & Hahn, C. E. (2003). Mathematical modelling of pulmonary gas transport. *Journal of mathematical biology*, *47*(1), 79-99.